

5 Chapter 5 The APL Program "OPTIMA"

This chapter consists of the listings of the APL functions which comprise the program OPTIMA. OPTIMA is used for minimization of a Boolean function to the minimal or the optimum EIT or ITE form.

It is used for combinational circuits of single or multiple outputs.

OPTIMA is based upon the previously-published theorems of mutual term exclusivity.

```
▽ AMPL PRI;X;LIM;U;TRA;Y;TRM;Z
[1] RRR←TTT←SSS←10
[2] TTT←TTT,G,AA,PRI
[3] SSS←SSS,COST PRI
[4] Z←FUND PRI
[5] RRR←RRR,(-YY+1),(Z∈DOT)/Z
[6] YY←YY+1
[7] →(0=Y←+/X←(PRI=0)^(FCT<0))/0
[8] TRA←1+X
[9] LIM←2*Y
[10] U←1
[11] A1:TRM←PRI+2*(TRA↑U)
[12] TTT←TTT,G,AA,TRM
[13] SSS←SSS,COST TRM
[14] Z←FUND TRM
[15] RRR←RRR,(-YY+1),(Z∈DOT)/Z
[16] YY←YY+1
[17] →(LIM>U←U+1)/A1
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▽

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▽ BRANCH
[1] ⍝ BRANCHING CONTROL
[2] 'BRANCHING POINT CHOSEN IS: ';PIV←MTR[1]
[3] FUN←FUND SMB←SBOL PIV
[4] ZCV←(FLS∈FUN)/FLS
[5] PRIMS 0
[6] QRS←PRS
[7] QJ←PJ
[8] 'TABLE OF BRANCHING TERMS:'
[9] (⊖QJ) PRINT(,QRS)
[10] 'INTEGERS AT THE RIGHT HAND SIDE ARE SO CALLED:'
    ROW NUMBERS.'
[11] 'CALL TRIAL.'
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      ▽ COMB;BMB;P;U
[1]  A SEQUENCE AMB IS MONOTONOUS IN BIT COUNT
[2]  BMB←AMB+2*( $\bar{1}+(1N)$ )
[3]  P←1
[4]  H1:CMB←10
[5]  U←P
[6]  H2:CMB←CMB,(((2*U)+(((2*U)>BMB)/BMB)))
[7]  →(N>U+U+1)/H2
[8]  AMB←AMB,CMB
[9]  BMB←CMB
[10] →(N>P+P+1)/H1
      ▽

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      ▽ R←COST X
[1]  R←+/( $(FCT>0)^{(1<X)}$ )-( $(FCT<0)^{(1<X)^{(0=2|X)}$ )
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      ▽ R←CPU
[1]  R←,' $\Delta$ CPU= $\square$ ,Q $\square$  60TH' $\square$ S $\square$ LI20' $\Delta$ FMT| $\Delta$ 21-I21
[2]   $\Delta$ 21←I21
      ▽

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▽ *Comment*

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[1]  'THIS PROGRAM IS USED FOR COMPUTER AIDED DESIGN
      OF SINGLE AND MULTIPLE OUTPUT COMBINATIONAL CIRCUITS.
      TO BEGIN CALL: DESIGN.'
[2]  ''
[3]  'REFERENCE: SVOBODA A., THE CONCEPT OF TERM
      EXCLUSIVENESS AND ITS EFFECT ON THE THEORY OF
      BOOLEAN FUNCTIONS. JOURNAL OF THE ASSOCIATION
      FOR COMPUTING MACHINERY, VOL. 22, NO. 3, JULY 1975.
      DE VRIES AND SVOBODA, MULTIPLE OUTPUT
      MINIMIZATION WITH MOSAICS OF BOOLEAN FUNCTIONS,
      IEEE TRANSACTIONS ON COMPUTERS, VOL. C-24, NO. 8,
      AUG. 75. '

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      ▽ R←DEFIN
[1]  FLT:NONE←R←10
[2]  ADD:R←R, $\square$ 
[3]  'TYPE: OK OR FLT OR ADD:'
[4]  → $\square$ 
[5]  OK:→0
      ▽

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      ▽ DESIGN;U;HR;VT;SET;RED
[1]  A FIRST PROCEDURE TO BE CALLED.
[2]  ABE←' AABBCDDDEEFFGGHHJJKKLLMMNN'
[3]  'NUMBER OF INDEPENDENT VARIABLES OF GIVEN BOOLEAN
      FUNCTIONS:'
[4]  HR←2*HOR←N←□
[5]  'THEIR SYMBOLS: ';ABE[1+2×\HOR]
[6]  'SYMBOLISM EXPLANATION:
      OK MEANS:NO MISTAKE
      FLT MEANS: FAULTY TYPING, REQUEST FOR RETYPING
      ADD MEANS: REQUEST FOR ADDITIONAL DATA INSERTION.'
[7]  ''
[8]  'TYPE THE NUMBER OF OUTPUTS.(BY TYPING: 1 THE
      PROCEDURE IS REDUCED TO AN N-MINIMIZATION OF A
      SINGLE GIVEN FUNCTION.)'
[9]  →(2=VT←2*VRT←□)/F7
[10] FCT←(VRTρ1), (HORρ1)
[11] MLT←1
[12] SET←(VRT,HR)ρ0
[13] U←1
[14] 'DEFINE FUNCTIONS BY DECIMAL EQUIVALENTS. TYPE:
      NONE FOR EMPTY SETS.'
[15] F1:'FUNCTION LABELED ';U;' IS TRUE AT:'
[16] TRU←DEFIN
[17] SET[U;1+TRU]←2
[18] 'FUNCTION NUMBER ';U;' IS UNSPECIFIED AT:'
[19] TRU←DEFIN
[20] SET[U;1+TRU]←1
[21] →(VRT≥U+U+1)/F1

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[22] RED←(HRρ1),(0<SET[1;])
[23] U←1
[24] F2:RUD←(HR×2*U)ρ(0<SET[U+1;])
[25] RED←RED,(RED^RUD)
[26] →(VRT>U+U+1)/F2
[27] RED←(VT,HR)ρRED
[28] U←0
[29] F3:RED[1+(2*U);]←SET[(U+1);]
[30] →(VRT>U+U+1)/F3
[31] TRU←(,RED=2)/-1+1HR×VT
[32] FLS←(,RED=0)/-1+1HR×VT
[33] VV←VRT
[34] →(0<N+HOR+VRT)/F4
[35] F7:MLT←0
[36] FCT←Nρ1
[37] 'TYPE-IN DECIMAL EQUIVALENTS OF TRUE MINTERMS
OR THE SYMBOL OF THE CORRESPONDING VECTOR.'
[38] TRU←DEFIN
[39] 'TYPE IN UNSPECIFIED MINTERMS (IF NONE, TYPE:
NONE).'
[40] FLS←TRU,FLS←DEFIN
[41] FLS←(∼((-1+12*N)∈FLS))/-1+12*N
[42] VV←[0.5×N
[43] F4:MINIMUM
▽

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```

▽ EXAMPLE;W;ALL;Q;JDI
[1] 'TYPE NUMBER OF VARIABLES: ' *
0 0 ρ6I1,60Iϕ 60 60 60 60 TI20
[2] ALL←2*□
[3] MINS←DONTS←1Q←0
[4] JDI←+/? 2 2
[5] →2+2×JDI
[6] MINS←MINS,Q
[7] →10
[8] DONTS←DONTS,Q
[9] →10
[10] →(ALL>Q←Q+1)/4
[11] W←MINS
[12] 'MINS→→→→';MINS←DONTS
[13] 'DONTS→→→→';DONTS←W
▽

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      ▽ EXTEND
[1]  GU← 5 2
[2]  L4:T←0
[3]  L2:→(0<ρTRU)/L3
[4]  →(0<GOT+1)/LL
[5]  L3:→((ρTRU)≥T←T+1)/L5
[6]  →(0<GOT←2)/LL
[7]  L5:AA←A←TRU[T]
[8]  SMB←SBOL TRU[T]
[9]  WT←+/2>SMB
[10] →(WT≤CLO)/HOP
[11] →(0<GOT←3)/LL
[12] HOP:FUN←FUND SMB
[13] DCV←(FUN∈TRU)/FUN
[14] ZCV←(FLS∈FUN)/FLS
[15] PRIMS 0
[16] DEL←FULSET
[17] LIST MLT
[18] →GU[NEXT]
[19] LL:→0
      ▽

      ▽ FORWARD
[1]  MTR←TRU
[2]  MST←MST,SYM,SET
[3]  CRS←CRS,PIV,CRI
[4]  'NEW'
[5]  BRANCH
[6]  →0
      ▽

      ▽ DEL←FULSET;BNC;B;RED;X;XTR
[1]  XTR←,DCV[1]
[2]  DEL←PJρ0
[3]  ALL←PRS>3
[4]  BNC←((Nρ2)τDCV[1])
[5]  →(1=B←ρDCV)/0
[6]  J←2
[7]  G2:RED←(PJ,N)ρ(((Nρ2)τDCV[J])≠BNC)
[8]  RED←RED^ALL
[9]  →(0=X←~^/∨/RED)/G1
[10] XTR←XTR,DCV[J]
[11] DEL←DEL∨(∨/RED)×X
[12] G1:→(B≥J←J+1)/G2
[13] DCV←XTR
      ▽

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      ▽ FUN←FUND S;U;BNC
[1]  BNC←2|S
[2]  FUN←1ρ(2⊥BNC)
[3]  U←0
[4]  →(N<U+U+1)/0
[5]  →(1<S[U])/4
[6]  FUN←FUN,(FUN+(1-2×BNC[U])×2*N-U)
[7]  →4
      ▽

      ▽ GRAF;TEX;P;V;U;X;Y
[1]  TEX←(8×VRT+HOR)ρ' '
[2]  TEX[-2+4×(⊥(2×VRT+HOR))]-←ABE[1+(⊥(2×VRT+HOR))]-]
[3]  TEX[(-6+8×HOR)+(8×⊥VRT)]←' '
[4]  TEX
[5]  ' '
[6]  P←1
[7]  S6:LIN←PSM[P;]
[8]  TEX←⊥0
[9]  V←1
[10] S5:Y←2|X←LIN[V]
[11] U←1
[12] S4:→((U≠2)^(U≠6))/S1
[13] →(V>HOR)/S7
[14] →(2>X)/S2
[15] →(2≠5|U+Y)/S2
[16] TEX←TEX,'□'
[17] →S3
[18] S2:TEX←TEX,'|'
[19] S3:→(8≥U+U+1)/S4
[20] →(N≥V+V+1)/S5
[21] TEX
[22] →(CTR≥P+P+1)/S6
[23] →30
[24] S1:TEX←TEX,'-'
[25] →S3
[26] S7:→(U≠6)/S1
[27] →((X≠0)^(Y=0))/S2
[28] TEX←TEX,'○'
[29] →S3
[30] SSM←+ / + / (1<PSM)×((CTR,N)ρ(ΦFCT))
[31] ' '
[32] 'S-VALUE OF THE CIRCUIT: ' ;SSM
      ▽

      ▽ R←IMPAS
[1]  S←+ / (1<PRS)×((PJ,N)ρFCT)
[2]  R←L/S
      ▽

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    ▽ INDX;G;H;X;CNT;SCL;NMN;SUT
[1]  NMN←ρMNR
[2]  H←LYM+X+1
[3]  N2: CNT←TON[H]
[4]  SCL←-1+ιCNT
[5]  G←1
[6]  N1: SUT←MNR[X]+ι(MNR[X+1]-MNR[X]+1)
[7]  IND[SUT]←SCL[G]
[8]  IMD[SUT]←H
[9]  →(NMN≤X+X+1)/0
[10] →(CNT≥G+G+1)/N1
[11] →(0<H+H-1)/N2
    ▽

    ▽ LIST HOP;PJ;QRS;S;NEG;POS;RED
[1]  →(0<PJ←+/~DEL)/L1
[2]  P8:→(0<NEXT←1)/0
[3]  L1: PRS←(~DEL)÷PRS
[4]  →(0=HOP)/P7
[5]  RED←⊖(Nρ2)∇DCV
[6]  NEG←~∇÷RED
[7]  POS←~∇÷~RED
[8]  RED←(PJ,N)ρ(2×(NEG∇POS)^(FCT<0)^(~2|PRS[1;]))
[9]  PRS←PRS+2×RED
[10] P7: DR←IMPAS
[11] →((BAR<DR)^(HOP=0))/P8
[12] PRS←(DR=S)÷PRS
[13] STOR PRS[1;]
[14] NEXT←2
    ▽

    ▽ MARQ;RED
[1]  RED←(2*N+1)ρ' * '
[2]  RED[(2×TRU)+1]←'1'
[3]  RED[(2×FLS)+1]←'0'
[4]  □←((2*VV),(2×2*(N-VV)))ρRED
[5]  ' '
    ▽

    ▽ MINIMUM;X
[1]  CLO←CTR←0
[2]  PRM←CVR+SET←CRI+ι0
[3]  COMB
[4]  ORDER
[5]  ' '
[6]  'MARQUAND CHARTS WILL BE USED.'
[7]  ' '

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[8]  '  CONSULT THE WEIGHT TABLE TO ESTIMATE THE
      DIFFICULTY OF THE PROBLEM. FOR A FUNCTION OF MORE
      THAN 6 VARIABLES WHICH HAPPENS TO HAVE ONLY A
      SMALL COUNT OF LOW WEIGHT MINTERMS (W=0,1,2,3) IT
      IS ADVISABLE TO BEGIN WITH THE LOWEST WEIGHT VALUE
      FOUND IN THE TABLE AS A LIMIT, SET FOR THE EXECUTION.
      THIS LIMIT CAN BE INCREASED LATER ONE BY ONE WHEN
      IT BECOMES CLEAR THAT THE GENERATION OF NEW TERMS
      IS ADEQUATE AND THE CORRESPONDING EXECUTION TIME
      IS ACCEPTABLE.'
[9]  GO← 16 21 10
[10] 'STATE OF THE CRITICAL SET: ' ;CRI
[11] '' * 'SET A LIMIT FOR W OR TERMINATE BY TYPING:
      0'
[12] →(CLO≥X+□)/END * 'PRESENT STATE: ' * MARQ
[13] CLO←X
[14] EXTEND
[15] →GO[GOT]
[16] WON: 'N-MINIMAL FORM: '
[17] (,CRI) PRINT SET
[18] →(O=MLT)/O

[19] 'YOU MAY CALL SCHEMATIC. THE RESULTING GRAPH
      DESCRIBES THE CIRCUIT UNDER THE FOLLOWING RULES:
          HORIZONTALLY ALIGNED QUADS (---□---) REPRESENT
          THE INPUTS OF AN AND-GATE
          HORIZONTALLY ALIGNED CIRCLES (---○---) REPRESENT
          AN INPUT OF AN OR-GATE WHOSE OUTPUT VARIABLE IS
          DESIGNATED BY A LITERAL PRINTED ABOVE THE CIRCULAR
          MARKER ○ .'
[20] →0
[21] 'CYCLIC. PARTIAL FORM: '
[22] (,CRI) PRINT SET
[23] MLUV
[24] 'RESIDUAL FUNCTION: '
[25] MARQ
[26] GRS←CRI
[27] MST←SET
[28] MTR←TRU
[29] 'TO CONTINUE CALL BRANCH'
[30] →0
[31] END: 'PARTIAL RESULT: '
[32] (,CRI) PRINT SET
[33] MLUV
[34] MARQ
[35] →0

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THE APL PROGRAM "OPTIMA"

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▽ MLUV
[1] 'RIGHT HAND SIDE BORDER INTEGERS
    BELONG TO A CRITICAL SET.'
▽

▽ OPTIMUM;GRP;S;ALL;TER;H;X;MEZ;LIM;TRN;G;SPL;MAX;
  CST;R;GRU;NR;MN;MX;XST;MZ;TRY;LM;TAK;V;MNO;D;TWN;
  E;HOP
[1] R RIGOROUS MULTIPLE OUTPUT CIRCUIT DESIGN
    OPTIMIZATION.
[2] →((ρDOT)=ρCRI)/Q9
[3] GRP←0.5×ρCVR
[4] SET←R←ALL←S←10
[5] YY←X←G←0
[6] Q1:AA←CVR[1+2×G]
[7] H←0
[8] MEZ←CVR[2×1+G]
[9] Q2:TER←PRM[(N×X)+1N]
[10] X←X+1
[11] AMPL TER
[12] ALL←ALL,TTT
[13] S←S,SSS
[14] R←R,RRR
[15] →(MEZ>H←H+1)/Q2
[16] →(GRP>G←G+1)/Q1
[17] R←R,(-YY+1)
[18] NR←ρR
[19] GRU←ρS
[20] TER←(GRU,(N+2))ρALL
[21] LYM←LIM←[ /IND←TER[;1]
[22] TRN←(LIM+1)ρ1
[23] G←0
[24] Q3:TRN[LIM+1-G]←+/INDεG
[25] →(LIM≥G←G+1)/Q3
[26] SPL←IND1Φ(1+1LIM+1)
[27] MAX←×/TON←TRN
[28] MNR←(R<0)/1NR
[29] IND←IMD←R
[30] INDX
[31] MST←(~(DOTεCRI))/DOT
[32] MEZ←ρMST
[33] ORG←FEW←FEU←10
[34] V←1
[35] S0:X←MST[V]
[36] DEX←(RεX)/1NR
[37] FEW←FEW,IND[DEX].
[38] ORG←ORG,((ρFEW)+1)
[39] FEU←FEU,IMD[DEX]
[40] →(MEZ≥V←V+1)/S0

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[41] MZ←ρORG
[42] DST←ρFEW
[43] MAT←(DST,(ρTRN))ρ-1
[44] V←1
[45] S1:MAT[V;FEU[V]]←FEU[V]
[46] →(DST≥V+V+1)/S1
[47] V←0
[48] S6:W←X+1
[49] VV←TRN+V
[50] →(SSM≤SS←+/S[SPL+VV])/GO
[51] S3:→(0<+/(MAT[X;]=VV))/S2
[52] →(ORG[W]>X+X+1)/S3
[53] GO:→(MAX>V+V+1)/S6
[54] Q9:'ABSOLUTE OPTIMUM REACHED.'
[55] →0
[56] S2:X←ORG[W]
[57] →(MZ≥W+W+1)/S3
[58] SSM←SS
[59] 'S-VALUE HAS BEEN REDUCED TO: ';SSM
[60] TAK←GRUρ0
[61] TAK[SPL+VV]←1
[62] SET←TAK/TER
[63] CRI←SET[;2]
[64] SET←(0,0,(Nρ1))/[2] SET
[65] CRI PRINT(,SET)
[66] 'AND THE CIRCUIT TO:'
[67] ''
[68] GRAF
[69] 'TYPE: GO IF YOU WANT TO CONTINUE. IF NOT, TYPE:
STOP.'
[70] →□
[71] STOP:→0
▽

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▽ ORDER;T;IND;A;LAB;RED;NTR;U;UU
[1] * WEIGHT ORDERING.
[2] NTR←ρTRU
[3] LAB←(N+1)ρ0
[4] RED←NTRρ0
[5] T←1
[6] G1:A←TRU[T]
[7] SMB←SBOL A
[8] WT←(+/2>SMB)
[9] IND←+/LAB[ι(WT+1)]
[10] LAB[WT+1]←LAB[WT+1]+1
[11] RED[(NTR+1)-(ι(NTR-(IND+1)))]←RED[NTR-(ι(NTR-
(IND+1)))]
[12] RED[IND+1]←A
[13] →(NTR≥T+T+1)/G1

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[14] DOT←TRU←RED
[15] 'WEIGHT TABLE:'
[16] ''
[17] T←U←1
[18] G3:→(0=UU←LAB[T])/G2
[19] 'W=';T-1;' FOR MINS: ';TRU[(U-1)+1UU]
[20] U←U+UU
[21] G2:→((N+1)≥T+T+1)/G3
[22] ''

```

▽

```

▽ PRIMS HOP;B;C;J;JJ;RED;RAD;ROD;LIM
[1] n PRIME IMPLICANTS INCIDENT WITH A POINT.
[2] TRS←10
[3] →(0<B←ρZCV)/P0
[4] TRS←TRS,SMB
[5] →P5
[6] P0:RED←Q(Nρ2)T ZCV
[7] RAD←(B,N)ρ(2|SMB)
[8] RAD←RED=RAD
[9] TRF←1+(SMB<2)
[10] CMB←((2*WT)>AMB)/AMB
[11] P1:JJ←TRF∩CMB[1]
[12] CMB←(~(CMB∈CMB[1]))/CMB
[13] C←Bρ(+/JJ)
[14] ROD←(B,N)ρJJ
[15] ROD←ROD∧RAD
[16] →(0≠+/(C=+[2] ROD))/P4
[17] TRS←TRS,(SMB+(4×JJ))
[18] →(0<HOP)/P4
[19] →(0=LIM←ρCMB)/P5
[20] RED←10
[21] J←1
[22] P3:→(0=+/JJ≠(JJ×(TRF∩CMB[J])))/P2
[23] RED←RED,CMB[J]
[24] P2:→(LIM≥J←J+1)/P3
[25] CMB←RED
[26] P4:→(0<ρCMB)/P1
[27] P5:QJ←PJ←(ρTRS)÷N
[28] PRS←(PJ,N)ρTRS
[29] BAR←IMPAS

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▽

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      ▽ IND PRINT SYM;SEP;NDX;P
[1]  →(0=ρIND)/0
[2]  CTR←(ρSYM)÷N
[3]  NDX←(CTR,N)ρ((2×1N)-1)
[4]  PSM+φ((CTR,N)ρSYM)
[5]  SEP←1+(PSM>1)×(NDX+(2|PSM))
[6]  P←1
[7]  NXT:ABE[SEP[P;]]; ' ' ;IND[P]
[8]  ' '
[9]  →(CTR≥P←P+1)/NXT
      ▽

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      ▽ R←SBOL A;U
[1]  R←BNC+(Nρ2)τA
[2]  U←1
[3]  →((A+(1-2×BNC[U])×2*N-U)∈FLS)/5
[4]  →6
[5]  R[U]←2+R[U]
[6]  →(N≥U←U+1)/3
      ▽

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      ▽ SCHEMATIC
[1]  'FIRST APPROXIMATION OF THE OPTIMAL NETWORK: ' ; ' '
[2]  GRAF
[3]  'CALL OPTIMUM.'
[4]  →0
      ▽

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```

      ▽ STOR SMB
[1]  SET←SET,SMB
[2]  CRI←CRI,AA
[3]  TRU←TRU[(~(TRU∈DCV))/1ρTRU]
[4]  →(MLT=0)/0
[5]  CVR←CVR,AA,QJ
[6]  PRM←PRM,TRS
      ▽

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▽ TRIAL
[1]  ⍎ BRANCH TRIAL
[2]  'SELECT BRANCHING TERM BY TYPING ITS ROW NUMBER.'
[3]  →(QJ<X←⍋)/2
[4]  FUN←FUND SYM←QRS[X;]
[5]  DCV←(TRU∈FUN)/TRU←MTR
[6]  'BRANCHING TERM:'
[7]  (,PIV) PRINT SYM
[8]  TRU←TRU[(~(TRU∈DCV))/⍋TRU]
[9]  CLO←N
[10] SET←CRI←⍋0
[11] EXTEND
[12] →((GOT=1),(GOT≠1))/ 13 18
[13] 'COMPLETE COVERAGE. RESULTING FORM:'
[14] (CRS,PIV,CRI) PRINT(MST,SYM,SET)
[15] MLUV
[16] 'LITERALS TOTAL: ';+/1<MST,SYM,SET
[17] 'MAY CALL TRIAL AGAIN.'
[18] →0
[19] 'INCOMPLETE COVERAGE. PARTIAL FORM:'
[20] (CRS,PIV,CRI) PRINT(MST,SYM,SET)
[21] MLUV
[22] 'LITERALS TOTAL: ';+/1<MST,SYM,SET
[23] 'MAY CALL TRIAL AGAIN TO EXHAUST ALL POSSIBILITIES.
      IF THE FUNCTION IS TOO COMPLEX TO CONTINUE
      EXHUASTIVELY TOWARDS THE MINIMAL COUNT OF LITERALS,
      YOU CAN GIVE UP THE LITERAL MINIMIZATION BY CALLING:
      FORWARD.'
▽

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